Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

EE3901 Sensor Technologies

Formal Exams SP1/5, 2021

Examination

College of Science & Engineering

Examination Duration:	120 minutes
Reading Time:	15 minutes

Exam Conditions:

This is a Restricted Book Exam - see permitted materials

Exam paper will be released to the Library after the Supp/Def exam period

Exam paper may be taken by the student

Students to answer in 1 answer book

N/A - this is not an assignment

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculator - Graphics

Calculator - Non Programmable

Calculator - Scientific

Dictionary - Bilingual

Dictionary - English

Materials To Be Supplied To Students:

Answer Book/s

Instructions To Students:

Answer all questions. The total number of marks is 100.

Equation sheet for Sensor Technologies exam

1. Error, signal-to-noise ratio and dynamic range:

$$\text{absolute error} = (\text{measured value}) - (\text{true value})$$
$$\text{relative error} = \frac{(\text{measured value}) - (\text{true value})}{\text{true value}}$$
$$\text{SNR} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) = 20\log_{10}\left(\frac{M_{signal}}{M_{noise}}\right)$$
$$\text{DR} = 10\log_{10}\left(\frac{P_{max}}{P_{noise}}\right) = 20\log_{10}\left(\frac{M_{max}}{M_{noise}}\right)$$

2. Propagating variance of x through $y = f(x)$:

$$\sigma_y^2 = \left(\left.\frac{\partial f}{\partial x}\right|_{x_0}\right)^2 \sigma_x^2$$

3. Propagating multiple variances through $\boldsymbol{y} = f(\boldsymbol{x}) = f(x_1, ...)$:

$$\Sigma_y = J\Sigma_x J^T$$

where J is the Jacobian evaluated at the given $\boldsymbol{x}$ values:

$$J = \left[\begin{array}{ccc} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots \\ \vdots & \vdots & \ddots \end{array}\right]_{\boldsymbol{x}=\boldsymbol{x}_0} .$$

In the case that y is a scalar and all variables are uncorrelated,

$$\sigma_y = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \cdots}$$

4. Kalman filter:

$$\begin{aligned} \text{Predicted state:} \quad & \hat{\boldsymbol{x}}_{k|k-1} = F_k \hat{\boldsymbol{x}}_{k-1} + B_k \boldsymbol{u}_k \\ \text{Predicted cov.:} \quad & P_{k|k-1} = F_k P_{k-1} F_k^T + Q_k \\ \text{Measurement res.:} \quad & \boldsymbol{y}_k = \boldsymbol{z}_k - H_k \hat{\boldsymbol{x}}_{k|k-1} \\ \text{Measurement res. cov.:} \quad & S_k = H_k P_{k|k-1} H_k^T + R_k \\ \text{Kalman gain:} \quad & K_k = P_{k|k-1} H^T S_k^{-1} \\ \text{Updated state:} \quad & \hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + K_k \boldsymbol{y}_k \\ \text{Updated covariance:} \quad & P_k = \left(I - K_k H_k\right) P_{k|k-1}. \end{aligned}$$

5. Extended Kalman filter:

$$\begin{aligned} \text{Predicted state:} \quad & \hat{\boldsymbol{x}}_{k|k-1} = f\left(\hat{\boldsymbol{x}}_{k-1}\right) + b\left(\boldsymbol{u}_k\right) \\ \text{Measurement res.:} \quad & \boldsymbol{y}_k = \boldsymbol{z}_k - h\left(\hat{\boldsymbol{x}}_{k|k-1}\right) \end{aligned}$$

Let F_k and H_k be the Jacobians of f and h respectively.

6. Piezoresistive sensors (strain gauges):

$$\sigma = \frac{F}{A}, \quad \epsilon = \frac{\Delta L}{L_0}, \quad \sigma = E\epsilon, \quad \frac{\Delta R}{R_0} = G\epsilon$$

where σ is stress (N/m^2), F is force, A is area, ϵ is strain, L is length (m), E is Young's modulus, R is resistance and G is gauge factor.

7. Temperature coefficient of resistance:

$$\text{TCR} = \frac{\left(\frac{dR}{dT}\right)}{R}$$

8. Self-heating of RTDs:

$$\Delta T = \frac{P_D}{\delta}$$

where ΔT is the self-heating error, P_D is the power dissipated in the RTD and δ is the heat dissipation constant.

9. Resistance, capacitance and inductance:

$$\begin{aligned} R &= \frac{\rho L}{A} \\ C &= \frac{\epsilon A}{L}, \quad \epsilon = \epsilon_r \epsilon_0, \quad \epsilon_0 = 8.85 \text{ pF/m} \\ L &= \frac{\mu N^2 A}{L}, \quad \mu = \mu_r \mu_0, \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \end{aligned}$$

where ρ is resistivity, L is length or thickness, A is cross-sectional area, ϵ is permittivity, ϵ_r is relative permittivity, μ is permeability, and N is the number of wire turns in a solenoid.

10. Thermocouple voltage:

$$V = E(T_{sense}) - E(T_{ref})$$

where E is the thermocouple characteristic function.

11. Transimpedance amplifiers:

$$C_f = \sqrt{\frac{C_{in}}{\pi R_f f_T}}$$
$$f_{-3\text{dB}} \approx \sqrt{\frac{f_T}{2\pi R_f C_{in}}}$$

where C_f is the capacitance along the op-amp feedback path, C_{in} is the total capacitance at the op-amp input, R_f is the resistance along the op-amp feedback path, and f_T is the op-amp's gain-bandwidth product.

12. Piezoelectric sensors:

$$\begin{aligned} \text{Sensor equation: } & \boldsymbol{D} = [d]\,\boldsymbol{T} + [\epsilon]\,\boldsymbol{E} \\ \text{Actuator equation: } & \boldsymbol{S} = [d]^T\,\boldsymbol{E} + [s]\,\boldsymbol{T} \end{aligned}$$

where $\boldsymbol{D}$ is electric displacement, $[d]$ is the 3×6 piezoelectric charge coefficient matrix, $\boldsymbol{T}$ is stress, $\boldsymbol{E}$ is electric field, $\boldsymbol{S}$ is strain and $[s]$ is elastic compliance.

13. Hall effect sensors:

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}, \quad \boldsymbol{v} = \mu\boldsymbol{E}, \quad V_H = G_H \frac{iB}{qN_cL}$$

where $q = \pm 1.602 \times 10^{-19}$ C is the charge of a hole or electron, $\boldsymbol{v}$ is charge carrier velocity, $\boldsymbol{B}$ is the magnetic field vector and B is the component of magnetic field perpendicular to both the current and electric field, μ is mobility, $\boldsymbol{E}$ is electric field, $G_H \approx 0.7$ to 0.9 is the geometry factor, i is current, L is the thickness of the sensor (in the direction of $\boldsymbol{B}$), and N_c is the charge carrier density in the semiconductor.

14. Eddy current skin depth:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

where ρ is resistivity, $\omega = 2\pi f$ is natural frequency, and $\mu = \mu_r\mu_0$ is permeability.

15. First order RC filters:

$$f_{-3\text{dB}} = \frac{1}{2\pi RC}$$

Question 1

The transfer function of a resistive temperature detector (RTD) is

$$R(T) = 100(1 + \alpha_1 T + \alpha_2 T^2),$$

where R is resistance in ohms, T is temperature in °C, $\alpha_1 = 3.91 \times 10^{-3}\ (°\mathrm{C})^{-1}$ and $\alpha_2 = -5.775 \times 10^{-7}\ (°\mathrm{C})^{-2}$. The device has a heat dissipation constant of $\delta = 100\ \mathrm{mW/°C}$.

(a) Which variable is the measurand in this scenario? (2 marks)

(b) Find the sensitivity of this device at $T = 40$ °C. (2 marks)

(c) At the two temperature limits (0 °C and 100 °C), calculate the maximum current that can flow through the sensor in order to keep the self-heating error below 0.1 °C. Hence, choose a current limit that is suitable for the entire range. (4 marks)

(d) Sketch a four-wire measurement circuit that would suit this sensor. (2 marks)

Question 2

You are designing a device to monitor the power consumption of a circuit. You are measuring the voltage v and current i and then calculating power using $P = vi$.

Your measurements of voltage and current are subject to Gaussian noise. You have measured the noise characteristics and determined the following properties:

Standard deviation in voltage: $\sigma_v = 0.05$ V
Standard deviation in current: $\sigma_i = 0.002$ A
Correlation: $\rho_{vi} = 0.1$

(a) Write down the covariance matrix representing the measurement uncertainty. (5 marks)

(b) At a given moment, your measurements are $v = 3.68$ V and $i = 205$ mA. What is the standard deviation in power? (5 marks)

Question 3

You are implementing a sensor fusion scheme for a fixed-wing UAV (unoccupied aerial vehicle).

(a) You are deciding which of the following algorithms to implement: the Kalman filter, the extended Kalman filter, and the unscented Kalman filter. How would you decide between these options? Describe which characteristics of the system and/or sensors would determine which of these algorithms could be used? (5 marks)

(b) All types of Kalman filter require information about the process covariance and the measurement covariance. What is the role of these covariance matrices? How would the algorithm be affected if the process covariance was too large, or the measurement covariance was too large? (5 marks)

Question 4

(a) A 200 Ω strain gauge with gauge factor G=200 is attached to a steel beam that is supporting a load of 4 kN in tension. The steel beam has cross-sectional dimensions of 5 cm × 2 cm and a Young's modulus of 190 GPa. What is the total strain gauge resistance when the beam is loaded? (5 marks)

(b) The resistance of a strain gauge changes by 0.25% when a strain of 25×10^{-6} is applied. Calculate the gauge factor. (5 marks)

Question 5

A Wheatstone bridge circuit contains two resistive sensors and two fixed resistors, as shown in Figure 1. The sensors have a nominal resistance of R_0 at a temperature T_0. They also have a temperature coefficient of resistance (TCR) of α.

The active sensor is in the lower-right position of the bridge and has a transfer function of the form

$$R = R_0(1+x)\big(1+\alpha(T-T_0)\big),$$

where x is the desired measurement. Meanwhile the sensor in the upper-right position is not exposed to changes in the measurand, so it only experiences changes in temperature. This sensor is used to provide temperature compensation.

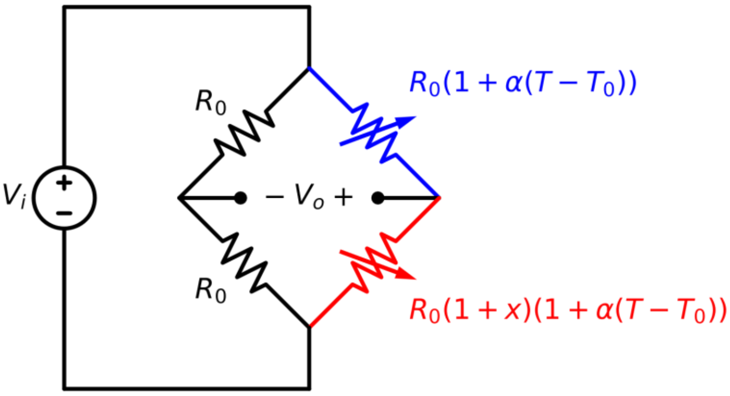

Figure 1: Temperature-compensated Wheatstone bridge circuit.

(a) Analyse this circuit and find the relationship between the output voltage V_0 and the measured signal x. (10 marks)

(b) List at least two ways to increase the sensitivity of this system. (5 marks)

Question 6

The capacitance sensor shown in Figure 2 measures the distance d between the probe and a conductive target.

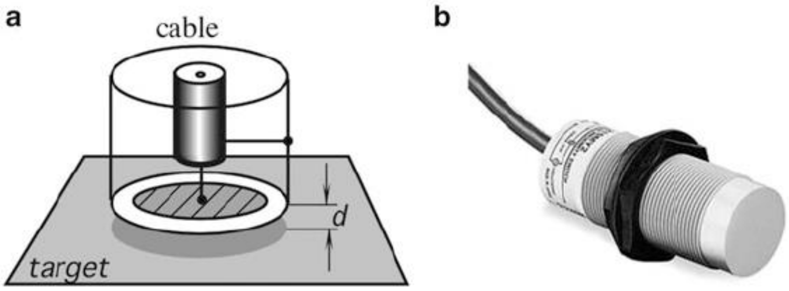

Figure 2: A capacitive probe. Figure from J. Fraden, Handbook of Modern Sensors, 5th ed., Springer, 2016.

(a) What is the expected relationship between the capacitance C and the measured distance? How does the sensitivity vary with distance? (5 marks)

(b) Sketch an interface circuit using a single op-amp that will generate an output voltage linearly proportional to the distance. (10 marks)

Question 7

The circuit diagram for a transimpedance amplifier is shown in Figure 3. You are using this type of interface circuit with an infrared photodiode that has a measured capacitance of 35 pF. The photodiode's datasheet claims a bandwidth of 5 MHz. The required amplification gain is 5 V/mA.

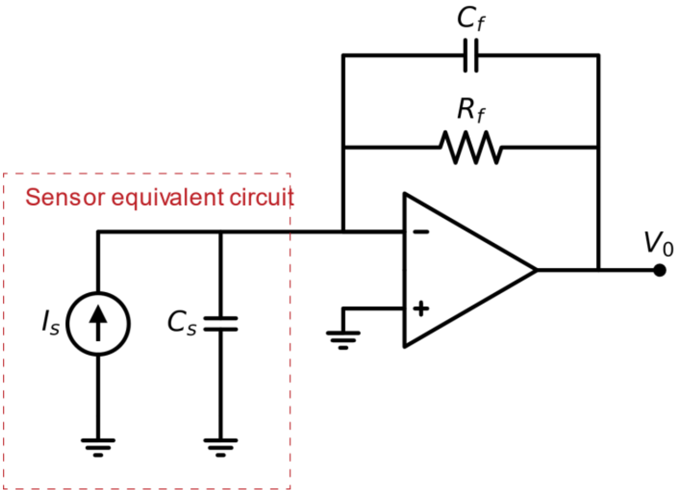

Figure 3: Transimpedance amplifier circuit.

(a) Specify the value of R_f so that the DC gain of the circuit matches the specification. (5 marks)

(b) What is the required op-amp specification such that the overall bandwidth is not limited by the amplifier? (5 marks)

Question 8

The circuits shown in Figure 4 use Type K thermocouples. An extract from the ITS-90 database for this type of thermocouple is shown below.

ITS-90 Table for type K thermocouple

Thermoelectric Voltage in mV

°C	0	1	2	3	4	5	6	7	8	9	10
0	0.000	0.039	0.079	0.119	0.158	0.198	0.238	0.277	0.317	0.357	0.397
10	0.397	0.437	0.477	0.517	0.557	0.597	0.637	0.677	0.718	0.758	0.798
20	0.798	0.838	0.879	0.919	0.960	1.000	1.041	1.081	1.122	1.163	1.203
30	1.203	1.244	1.285	1.326	1.366	1.407	1.448	1.489	1.530	1.571	1.612
40	1.612	1.653	1.694	1.735	1.776	1.817	1.858	1.899	1.941	1.982	2.023
50	2.023	2.064	2.106	2.147	2.188	2.230	2.271	2.312	2.354	2.395	2.436
60	2.436	2.478	2.519	2.561	2.602	2.644	2.685	2.727	2.768	2.810	2.851
70	2.851	2.893	2.934	2.976	3.017	3.059	3.100	3.142	3.184	3.225	3.267
80	3.267	3.308	3.350	3.391	3.433	3.474	3.516	3.557	3.599	3.640	3.682
90	3.682	3.723	3.765	3.806	3.848	3.889	3.931	3.972	4.013	4.055	4.096
100	4.096	4.138	4.179	4.220	4.262	4.303	4.344	4.385	4.427	4.468	4.509
110	4.509	4.550	4.591	4.633	4.674	4.715	4.756	4.797	4.838	4.879	4.920
120	4.920	4.961	5.002	5.043	5.084	5.124	5.165	5.206	5.247	5.288	5.328
130	5.328	5.369	5.410	5.450	5.491	5.532	5.572	5.613	5.653	5.694	5.735
140	5.735	5.775	5.815	5.856	5.896	5.937	5.977	6.017	6.058	6.098	6.138

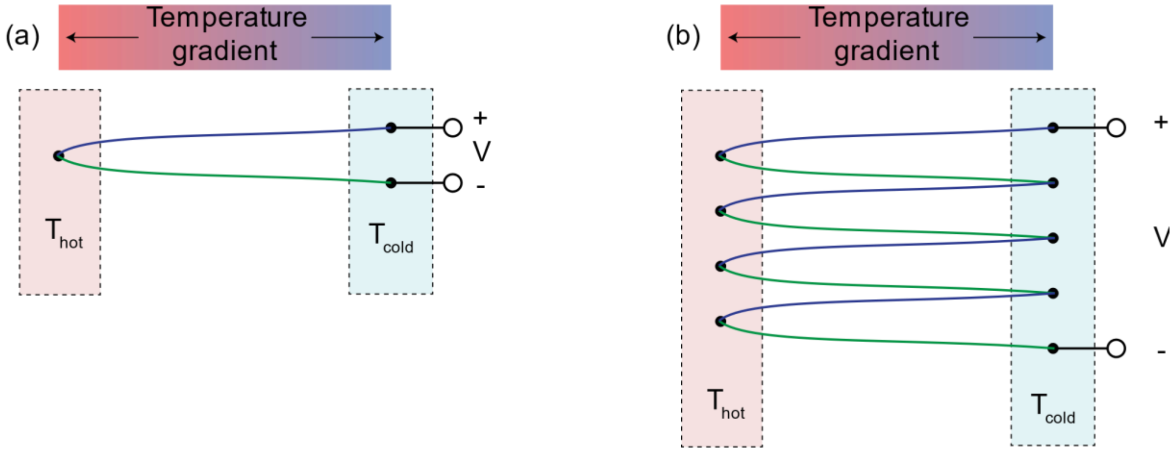

Figure 4: Thermocouple circuits.

(a) Referring to Figure 4 (a), the junction temperatures are $T_{hot} = 80$ °C and $T_{cold} = 25$ °C. What is the output voltage V? (5 marks)

(b) Figure 4 (b) shows the series connection of 4 identical thermocouples. If the junction temperatures are the same as before, what is the output voltage? (5 marks)

Question 9

A piezoelectric accelerometer and equivalent circuit model is shown in Figure 5. The material has a relative permittivity of $\epsilon_r = 1200$ and a resistivity of $\rho = 10^6$ Ω.m.

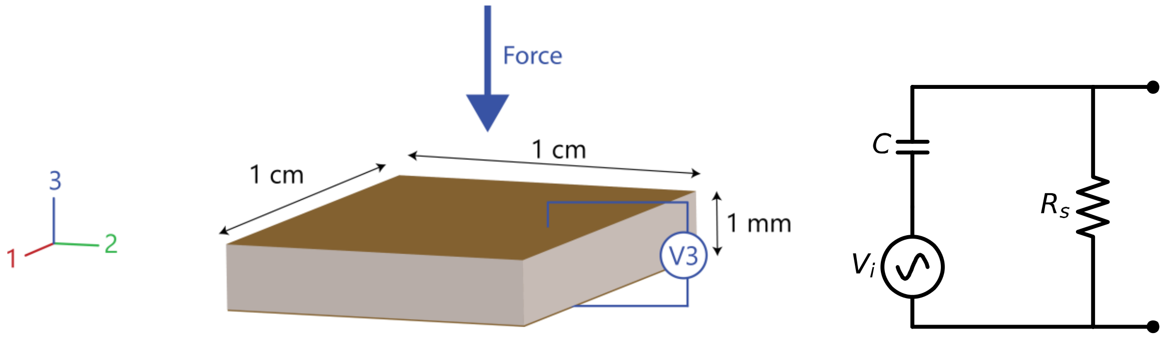

Figure 5: Piezoelectric element and equivalent circuit.

(a) The force is applied along axis **3**, and the voltage is measured along the same axis. Which piezoelectric coefficient (d_{ij}) controls the strength of the measured voltage? (2 marks)

(b) Given the material dimensions, calculate the geometric capacitance C and resistance R_s. (3 marks)

(c) Estimate the lowest frequency of operation based upon the highpass filter characteristics of the equivalent circuit. Sketch the expected shape of the frequency response curve (i.e. sketch output amplitude vs frequency). (5 marks)

END OF EXAMINATION