1. (a) The measurand is temperature.

(b) The sensitivity is

$$S = \frac{dR}{dT} = 100K_1 + 200K_2T$$

$$= 100 \times 3.91 \times 10^{-3} + 200 \times (-5.775 \times 10^{-7}) \times 40$$

$$= 0.3864 \ \Omega/^\circ C.$$

(c) At $T=0$, $R = 100\ \Omega$.

$$\Delta T = \frac{P_D}{\delta}$$

$$0.1 = \frac{I^2 \times 100}{0.1}$$

$$I^2 = 10^{-4}$$

$$I = 10 \text{ mA}.$$

At $T = 100$, $R = 100(1 + 3.91 \times 10^{-3} \times 100 - 5.775 \times 10^{-7} \times 100^2)$

$$= 138.5\ \Omega.$$

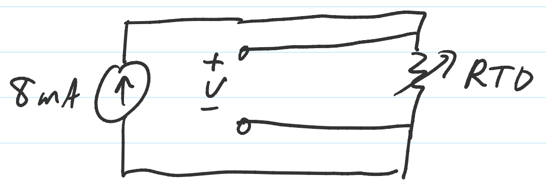
$$\Delta T = 0.1 = \frac{P_D}{\delta} = \frac{I^2 \times 138.5}{0.1}$$

$$I^2 = 72.202 \times 10^{-6}$$

$$I = 8.5 \text{ mA}.$$

$\therefore$ Must keep the current below 8.5 mA.

(d)



2. (a) $\Sigma_x = \begin{bmatrix} \sigma_v^2 & \rho\sigma_v\sigma_i \\ \rho\sigma_v\sigma_i & \sigma_i^2 \end{bmatrix} = \begin{bmatrix} 0.05^2 & 0.1 \times 0.05 \times 0.002 \\ 0.1 \times 0.05 \times 0.002 & 0.002^2 \end{bmatrix}$

$$= \begin{bmatrix} 2.5 \times 10^{-3} & 10 \times 10^{-6} \\ 10 \times 10^{-6} & 4 \times 10^{-6} \end{bmatrix}$$

(b) $J = \begin{bmatrix} \frac{\partial P}{\partial v} & \frac{\partial P}{\partial i} \end{bmatrix} = \begin{bmatrix} i & v \end{bmatrix} = \begin{bmatrix} 0.205 & 3.68 \end{bmatrix}$

$$\therefore \Sigma_y = J \Sigma_x J^T = \begin{bmatrix} 0.205 & 3.68 \end{bmatrix} \begin{bmatrix} 2.5 \times 10^{-3} & 10 \times 10^{-6} \\ 10 \times 10^{-6} & 4 \times 10^{-6} \end{bmatrix} \begin{bmatrix} 0.205 \\ 3.68 \end{bmatrix}$$

$$= \begin{bmatrix} 0.205 & 3.68 \end{bmatrix} \begin{bmatrix} 549.3 \times 10^{-6} \\ 16.77 \times 10^{-6} \end{bmatrix}$$

$$= 174.26 \times 10^{-6}$$

$$\therefore \quad \sigma_y = \sqrt{\Sigma_y} = 13.2 \text{ mW.}$$

3. (a) • Kalman filter: only applicable to systems with linear prediction models and linear measurement models.

• EKF: can handle non-linear systems or measurements, but must be differentiable.

• UKF: non-linear & non-differentiable. Also good when the derivatives are large/unstable/hard to calculate.

(b) These covariances are used to trade-off how much the filter believes the model vs how much it believes the measurements. Too large a process covariance means that the filter will track measurement noise instead of filtering it out. Too large a measurement covariance means that it will ignore measurements and just track the model.

4. (a) $\sigma = \frac{F}{A} = \frac{4000}{0.05 \times 0.02} = 4 \times 10^6 \ \frac{N}{m^2}.$

$$\varepsilon = \frac{\sigma}{E} = \frac{4 \times 10^6}{190 \times 10^9} = 2.1 \times 10^{-5}$$

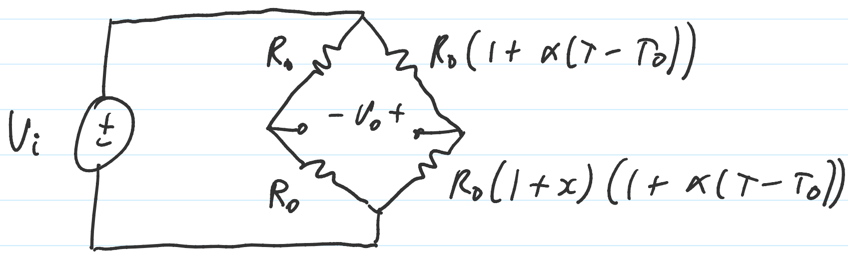
$$\frac{\Delta R}{R_0} = G\varepsilon = 200 \times 2.1 \times 10^{-5} = 0.0042$$

$$\therefore R = R_0 + \Delta R = 200(1 + 0.0042) = 200.84 \ \Omega$$

(b) $\frac{\Delta R}{R_0} = \frac{0.25}{100} = G\varepsilon = G \times 25 \times 10^{-6}$

$G = 100$

5. (a)



$$V_o = V_i\left(\frac{\cancel{R_0}\,(1+x)\,\cancel{(1+\alpha(T-T_0))}}{\cancel{R_0}\,\cancel{(1+\alpha(T-T_0))} + \cancel{R_0}\,(1+x)\,\cancel{(1+\alpha(T-T_0))}} - \frac{\cancel{R_0}}{2\cancel{R_0}}\right)$$

$$= V_i\left(\frac{1+x}{1+1+x} - \frac{1}{2}\right)$$

$$= V_i\left(\frac{1+x}{2+x} - \frac{1}{2}\right)$$

OK to stop here or simplify to:

$$= V_i\left(\frac{x}{2x+4}\right)$$

(b) Sensitivity could be increased by:

1. Upping V_i
2. Using an amplifier circuit.
3. Using a full bridge configuration.

6. (a) $C = \dfrac{\varepsilon A}{d}$ i.e. $C \sim \dfrac{1}{d}$.

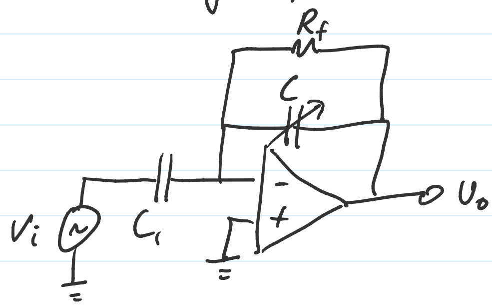
Inversely proportional to distance.

The sensitivity is higher when d is small.

(b) $Z_{sensor} = Z_s = \dfrac{1}{j\omega C} = \dfrac{d}{j\omega \varepsilon A}$

$\therefore$ Z_s is linearly proportional to d.

$\therefore$



Place the sensor in the feedback path.

$V_o \approx -\dfrac{Z_s}{Z_1} V_i$ i.e. proportional to d.

7. (a) $R_f = 5V/mA = 5\,000\ \Omega$.

(b) $f_{-3dB} = \sqrt{\dfrac{f_T}{2\pi R_f C_{in}}}$

Assume $C_{in} = 35$ pF only (neglect op-amp package).
Need $f_{3dB} \geq 5$ MHz.

$$\left(5 \times 10^6\right)^2 = \frac{f_T}{2\pi \times 5000 \times 35\times10^{-12}}$$

$$f_T = 27.49 \text{ MHz.}$$

$\therefore$ Choose an op-amp with GBP > 27.5 MHz.

8. (a) $V = 3.267 - 1 = 2.267$ mV.
(b) There are 4 series thermocouples so $V = 4 \times 2.267 = 9.068$ mV.

9. (a) In: $\underset{\sim}{D} = [g]\underset{\sim}{T} + \varepsilon \underset{\sim}{E}$
E_3 will be determined by

$$D_3 = \begin{bmatrix} g_{31} & g_{32} & g_{33} & g_{34} & g_{35} & g_{36} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ T_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \varepsilon E_3$$

$\therefore$ g_{33} is the parameter that influences V_3.

(b) $$C = \frac{\varepsilon_r \varepsilon_0 A}{L} = \frac{1200 \times 8.85\times10^{-12} \times (0.01)^2}{0.001} = 1.062 \text{ nF.}$$

$$R = \frac{\rho L}{A} = \frac{10^6 \times 0.001}{(0.01)^2} = 10 \text{ M}\Omega.$$

(c) $$f_{-3dB} = \frac{1}{2\pi RC} = 14.99 \text{ Hz}$$