Equation sheet for Sensor Technologies exam

1. Error, signal-to-noise ratio and dynamic range:

$$\text{absolute error} = (\text{measured value}) - (\text{true value})$$

$$\text{relative error} = \frac{(\text{measured value}) - (\text{true value})}{\text{true value}}$$

$$\text{SNR} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) = 20\log_{10}\left(\frac{M_{signal}}{M_{noise}}\right)$$

$$\text{DR} = 10\log_{10}\left(\frac{P_{max}}{P_{noise}}\right) = 20\log_{10}\left(\frac{M_{max}}{M_{noise}}\right)$$

2. Propagating variance of x through $y = f(x)$:

$$\sigma_y^2 = \left(\left.\frac{\partial f}{\partial x}\right|_{x_0}\right)^2 \sigma_x^2$$

3. Propagating multiple variances through a vector-valued function $\boldsymbol{y} = f(\boldsymbol{x}) = f(x_1, ...)$:

$$\Sigma_y = J\Sigma_x J^T$$

where J is the Jacobian evaluated at the given $\boldsymbol{x}$ values, Σ_y is the covariance of $\boldsymbol{y}$, and Σ_x is the covariance of $\boldsymbol{x}$. In the case that y is a scalar and all variables are uncorrelated,

$$\sigma_y = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \cdots}.$$

4. Jacobians:

$$J = \left[\begin{array}{ccc} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots \\ \vdots & \vdots & \ddots \end{array}\right]_{\boldsymbol{x}=\boldsymbol{x}_0}.$$

5. Covariance matrix for 2 variables x_1 and x_2:

$$\Sigma = \left[\begin{array}{cc} \sigma_{x_1}^2 & \rho\sigma_{x_1}\sigma_{x_2} \\ \rho\sigma_{x_1}\sigma_{x_2} & \sigma_{x_2}^2 \end{array}\right],$$

where σ is the standard deviation and ρ is the correlation.

6. Kalman filter:

$$\begin{aligned} \text{Predicted state: } & \hat{\boldsymbol{x}}_{k|k-1} = F_k\hat{\boldsymbol{x}}_{k-1} + B_k\boldsymbol{u}_k \\ \text{Predicted cov.: } & P_{k|k-1} = F_k P_{k-1} F_k^T + Q_k \\ \text{Measurement res.: } & \boldsymbol{y}_k = \boldsymbol{z}_k - H_k\hat{\boldsymbol{x}}_{k|k-1} \\ \text{Measurement res. cov.: } & S_k = H_k P_{k|k-1} H_k^T + R_k \\ \text{Kalman gain: } & K_k = P_{k|k-1} H^T S_k^{-1} \\ \text{Updated state: } & \hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + K_k\boldsymbol{y}_k \\ \text{Updated covariance: } & P_k = (I - K_k H_k) P_{k|k-1}. \end{aligned}$$

7. Extended Kalman filter:

$$\begin{aligned} \text{Predicted state: } & \hat{\boldsymbol{x}}_{k|k-1} = f(\hat{\boldsymbol{x}}_{k-1}) + b(\boldsymbol{u}_k) \\ \text{Measurement res.: } & \boldsymbol{y}_k = \boldsymbol{z}_k - h(\hat{\boldsymbol{x}}_{k|k-1}) \end{aligned}$$

Let F_k and H_k be the Jacobians of f and h respectively.

8. Piezoresistive sensors (strain gauges):

$$\sigma = \frac{F}{A}, \quad \epsilon = \frac{\Delta L}{L_0}, \quad \sigma = E\epsilon, \quad \frac{\Delta R}{R_0} = G\epsilon$$

where σ is stress (N/m^2), F is force, A is area, ϵ is strain, L is length (m), E is Young's modulus, R is resistance and G is gauge factor.

9. Temperature coefficient of resistance:

$$\text{TCR} = \frac{\left(\frac{dR}{dT}\right)}{R}$$

10. Self-heating of RTDs:

$$\Delta T = \frac{P_D}{\delta}$$

where ΔT is the self-heating error, P_D is the power dissipated in the RTD and δ is the heat dissipation constant.

11. Resistance, capacitance and inductance:

$$R = \frac{\rho l}{A}$$

$$C = \frac{\epsilon A}{d}, \quad \epsilon = \epsilon_r\epsilon_0, \quad \epsilon_0 = 8.85 \text{ pF/m}$$

$$L = \frac{\mu N^2 A}{l}, \quad \mu = \mu_r\mu_0, \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

where ρ is resistivity, l is length, A is cross-sectional area, ϵ is permittivity, ϵ_r is relative permittivity, d is distance, μ is permeability, and N is the number of wire turns in a solenoid.

12. Maxwell capacitance matrix:

$$\boldsymbol{Q} = C\boldsymbol{V}$$

where $\boldsymbol{Q} = \left(\begin{array}{ccc} Q_1 & \cdots & Q_N \end{array}\right)^T$ is the charge on each conductor and $\boldsymbol{V} = \left(\begin{array}{ccc} V_1 & \cdots & V_N \end{array}\right)^T$ is the voltage on each conductor.

13. Mutual capacitances can be calculated from the Maxwell capacitance matrix using

$$C_{m,ii} = C_{ii} - \sum_{\substack{k=1 \\ k\neq i}}^{N} C_{m,ik}$$

$$C_{m,ij} = -C_{ij}.$$

14. Thermocouple voltage:

$$V = E(T_{sense}) - E(T_{ref})$$

where E is the thermocouple characteristic function.

15. Transimpedance amplifiers:

$$C_f = \sqrt{\frac{C_{in}}{\pi R_f f_T}}$$
$$f_{-3\text{dB}} \approx \sqrt{\frac{f_T}{2\pi R_f C_{in}}}$$

where C_f is the capacitance along the op-amp feedback path, C_{in} is the total capacitance at the op-amp input, R_f is the resistance along the op-amp feedback path, and f_T is the op-amp's gain-bandwidth product.

16. Piezoelectric sensors:

$$\text{Sensor equation:} \quad \boldsymbol{D} = [d]\,\boldsymbol{T} + [\epsilon]\,\boldsymbol{E}$$
$$\text{Actuator equation:} \quad \boldsymbol{S} = [d]^T\,\boldsymbol{E} + [s]\,\boldsymbol{T}$$

where $\boldsymbol{D}$ is electric displacement, $[d]$ is the 3×6 piezoelectric charge coefficient matrix, $\boldsymbol{T}$ is stress, $\boldsymbol{E}$ is electric field, $\boldsymbol{S}$ is strain and $[s]$ is elastic compliance.

17. Hall effect sensors:

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}, \quad V_H = G_H \frac{iB}{qN_c L}$$

where $q = \pm 1.602 \times 10^{-19}$ C is the charge of a hole or electron, $\boldsymbol{v}$ is charge carrier velocity, $\boldsymbol{B}$ is the magnetic field vector and B is the component of magnetic field perpendicular to both the current and electric field, $G_H \approx 0.7$ to 0.9 is the geometry factor, i is current, L is the thickness of the sensor (in the direction of $\boldsymbol{B}$), and N_c is the charge carrier density in the semiconductor.

18. Eddy current skin depth:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

where ρ is resistivity, $\omega = 2\pi f$ is natural frequency, and $\mu = \mu_r \mu_0$ is permeability.

19. Linear variable differential transformers (LVDTs) have a transfer function of the form

$$V_0 = S V_i x$$

where V_0 is the output voltage, S is sensitivity, V_i is input voltage, and x is the displacement.

20. First order RC filters:

$$f_{-3\text{dB}} = \frac{1}{2\pi RC}$$